DECUS
PROGRAM LIBRARY

DECUS NO.	FOCAL8-50
TITLE	FOCAL VERSION OF RC ACTIVE FILTER
AUTHOR	Bean & Romans
COMPANY	University of Texas Southwestern Medical School Dallas, Texas
DATE	September 9, 1969
SOURCE LANGUAGE	FOCAL

Get something extra in filter design.

One BASIC program works for Butterworth and Chebyshev low-pass or high-pass RC-active circuits.

A time-shared computer program can do much more than free the engineer from the tedium of routine calculations in filter design.[1] A single program in BASIC,[2] derived from two fundamental equations, can be used to design Butterworth or Chebyshev filters, and either low-pass or high-pass versions of each.

The program, written in a language resembling simple English, determines the component values for N-pole filters. The design uses two-pole active sections with only Rs and Cs, no Ls, as the basic building blocks for higher-order filters. N-pole filters are created by cascading $N/2$ two-pole sections. The R and C values in each section are selected to achieve the desired filter response.

Low-pass and high-pass sections [3,4,5] used in the filters are shown in Fig. 1. Two capacitors, two resistors and a unity-gain active element (Table 1) serve to synthesize a complex pair of poles in the filter characteristic.

The filters described in this article are relatively inexpensive. They may be built as either discrete circuits or hybrid microcircuits and for either commercial or military use.

As hybrid microcircuit designs they possess the following advantages:

- Since they use no Ls, the resulting circuit is potentially smaller, more stable and has a higher Q at low frequencies than passive LC designs.
- The Cs can be chosen as standard values. Even though the Rs are non-standard, they are relatively easy to obtain.
- The frequency response can be adjusted by varying only the Rs. One filter can therefore be readily tuned into phase track with another, or trimmed to a given specification.

In addition this design approach:

- Uses a minimum of Rs and Cs to synthesize a two-pole function.
- Requires only one unity-gain active element for each filter section.
- Has low sensitivity to parameter changes.

Several of the many possible types of unity-gain active elements are shown in Table 1. The most important figure of merit for these voltage follower elements is their current gain β because accurate filter synthesis requires a high input impedance and low output impedance. The equations used to calculate the filter component values assume a perfect active element, $\beta = \infty$. In practice the active elements are imperfect, especially at higher frequencies. Finite input impedance causes insertion loss and frequency response distortion; non-zero output impedance causes reduced stop-band attenuation; and variations from unity-gain change the resonant response of the section.

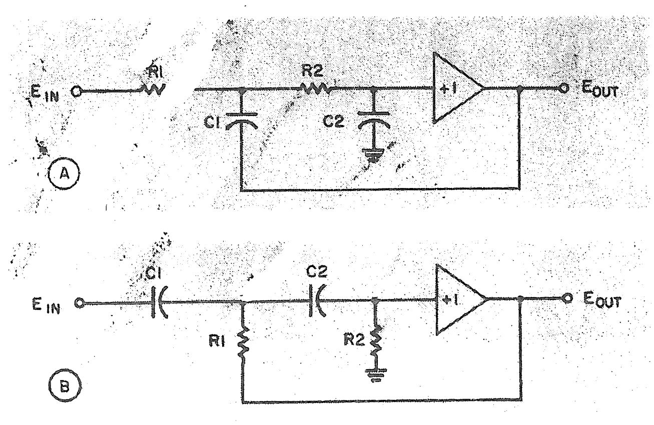
In view of these effects, it makes little sense to use 1% components to obtain a precise filter response, unless the input impedance of the active element is greater than $100R_2$ and the output impedance is less than $R_1/100$. It is also senseless to seek high stop-band attenuation in a frequency region where β is significantly decreased.

Align sections by adjusting only resistors

Component values (Fig. 1) for the basic low-pass or high-pass filter sections are computed from the equations for the pole locations $(\sigma_i + j\omega_i)$ of the normalized functions:

$$\sigma_i = X1 \cos (P_i)$$

$$j\omega_i = jX2 \sin (P_i).$$



1. **Two-pole circuit sections serve as basic building blocks** that are cascaded to form multiple-pole filters, low-pass section above, and high-pass section, below. The component values in each section are computer-selected (to obtain either a Butterworth or Chebyshev response).

Russell Kincaid and Frederick Shirley, Technical Staff, Sanders Associates, Inc., Nashua, N.H.

[illegible] 1, X_2 = 1, for Butterworth filters, [illegible]nd X_1 and X_2 depend on the passband ripple [illegible]nd number of poles for Chebyshev filters. The [illegible]uations for the component values are shown [illegible] Table 1.

The over-all filter design is not limited to a [illegible]rticular ratio of component values. Each two-[illegible]le section may therefore be independently [illegible]igned by adjusting only the two resistor values. [illegible]ree cases occur:

1. When the [illegible] tion are fairly close to the design values, the filter alignment can be improved by nearly an order of magnitude by adjusting only one of the two resistors so that the desired response is obtained at the cut frequency. For thick-film or potted sections, the adjustment may be made externally by adding a series trim resistor either to R_1 of the low-pass circuit or to R_2 of the high-pass circuit (Fig. 1).

2. When both C values are out of tolerance by comparable large percentages, the section may be aligned by a two-step procedure. First, impedance-scale the section by off-adjusting the two resistors by the same percentage as the capacitors but in the opposite direction. This will improve the frequency response of the section. It will modify the section impedance level to accommodate the varied C values. Second, trim the response of the section at the cut frequency by adjusting one of the two resistors, as previously described. It is not advisable to trim both resistors since their effects on the frequency response are interdependent.

3. When the two C values are out of tolerance by different large percentages, the section response may best be improved by off-adjusting the Rs to recomputed values. The revised R values are obtained by rerunning the computer program with the actual C values inserted in place of the nominal ones. One of the two resistors may then be trimmed, if desired.

Sensitivity influences filter response

If the circuit component values are out of tolerance—due to initial selection error, environmental variation or aging—the filter response will vary from nominal. The variation of filter gain $\Delta G/G$ with component value variation $\Delta V/V$ is determined by the sensitivity factor S:

$$\Delta G/G = S \times \Delta V/V.$$

If $S = 1$, the equation shows that a component variation of 1% is equivalent to 20 $\log_{10}$ (1-0.01), resulting in a gain variation of only 1 dB. For a large value of S—say, $S = 100$—the same equation shows that a component variation of 1% is equivalent to 100 × (0.01) = 1 (or a 100% increase). As the component varies by 1%, G

Table 1. Unity gain active elements

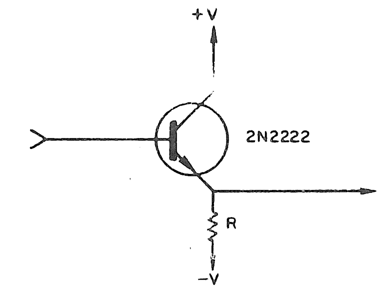
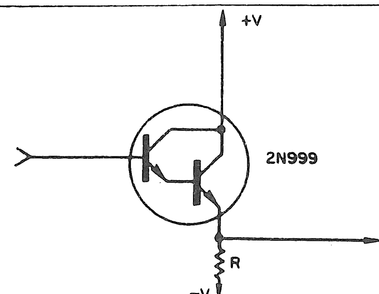
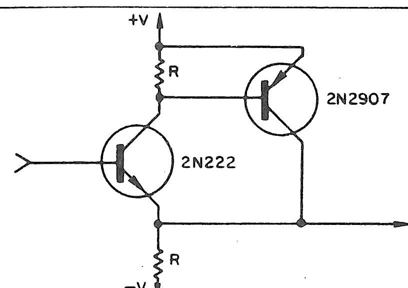
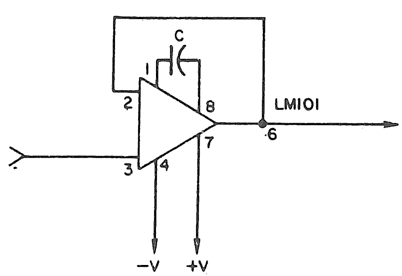
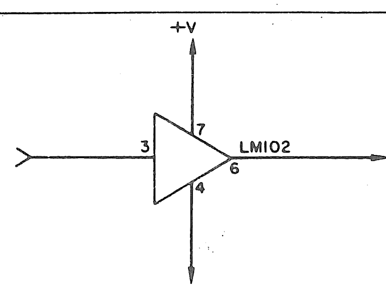
Circuit element	Approximate current gain (β) (at low frequencies)
 Simple emitter follower	100
 Darlington emitter follower	4000
 Compound transistor follower	4000
 Operational amplifier follower	>100,000
 Integrated circuit follower	>100,000

Table 2. Filter component value formulas*

Low-pass section		High-pass section	
Formula	**Eq. Nos. (see box)**	**Formula**	**Eq. Nos. (see box)**
$C_1 = C$	14a	$C_1 = C$	18a
$C_2 = C/M$	14b	$C_2 = C$	18b
$R_i = \frac{-\sigma_i}{2\pi F\ C}\left[1 + \left(1 - \frac{A_i}{\sigma_i^2 M}\right)^{\frac{1}{2}}\right]$	15a	$R_1 = \frac{-\sigma_i}{2\pi FC}$	19a
$R_i = \frac{-\sigma_i M}{2\pi FA_i C}\left[1 - \left(1 - \frac{A_i}{\sigma_i^2 M}\right)^{\frac{1}{2}}\right]$	15b	$R_2 = \frac{-A_i}{2\pi F\sigma_i C}$	19b
$M \geqslant A_i/\sigma_i^2$	16		

*Resistor values are scaled to cut frequency F.

increases 100%, or from G to $2G$, a gain variation of 6 dB ($20 \log_{10} 2 = 6$ dB).

Since a sensitivity factor is associated with each component, the worst-case variation for the complete section occurs when each component has a maximum error in an additive direction.

The sensitivity factors for components in the basic active filter sections (Fig. 1) vary with frequency and section Q. For the worst-case frequency, the sensitivity, S, for each of the R and C components (Fig. 1) is:

Q	S
1	0.01
100	0.3

and for variations in gain of the active element is

Q	S
1	0.02
5	1.0
8	100.0

The high-gain sensitivity factor is not harm-

```
0 DATA 2,4,1,2.4,10
10
20 REM     RC ACTIVE FILTER PROGRAM  BY R. KINCAID AND F. SHIRLEY
30 REM  SANDERS ASSOC. INC, NASHUA, N.H.  11/5/68
40 REM     LINE 0 SHOULD BE:  0 DATA T,N,F,R,C -- WHERE T IS THE TYPE
50 REM  (1=LOWPASS, 2=HIGHPASS);  N IS THE NO. OF POLES (MUST BE EVEN);
60 REM  F IS THE CUT FREQ IN KHZ;  R IS THE PASSBAND RIPPLE IN DB (0
70 REM  FOR BUTTERWORTH);  AND C IS THE MAX C IN NANOFARADS.
80
90 READ T,N,F,R,C
100 LET Q0=6.28318531
110 LET Q1=2.30258509
120 DIM F(37),G(37),C(25)
130 PRINT
140 PRINT
150 PRINT
160 PRINT
170 GOSUB 2020
180
190 REM   "I" ITERATION LOOP -- CALCULATIONS FOR EACH 2-POLE SECTION
200 FOR I=1 TO N/2
210 IF R>0 THEN 290
220
230 REM   BUTTERWORTH CIRCLE
240 LET X1=1
250 LET X2=1
260 GO TO 350
270
280 REM   CHEBYSHEV ELLIPSE
290 LET E=SQR(10↑(R/10)-1)
300 LET D=(1/E+SQR(1/E/E+1))↑(1/N)
310 LET X1=(D-1/D)/2
320 LET X2=(D+1/D)/2
330
340 REM   ROOT PAIR LOCATIONS
350 LET P=Q0/4+Q0/4/N*(2*I-1)
360 LET S=X1*COS(P)
370 LET W=X2*SIN(P)
380 LET A=S*S+W*W
390 LET R0=-S/Q0/F/C*1E3
400 LET C1=C
410 IF T>1 THEN 650
420
430 REM   LOWPASS C2 VALUE
440 IF I>1 THEN 500
450 DATA 6, 1.0, 1.5, 2.2, 3.3, 4.7, 6.8
460 READ K6
470 FOR K=1 TO K6
480 READ C(K)
490 NEXT K
500 LET N2=INT(LOG(C)/Q1)
510 FOR N1=-1 TO 99
520 FOR K=K6 TO 1 STEP -1
530 LET C2=C(K)*10↑(N2-N1)
540 LET M=C/C2
550 IF M>A/S/S THEN 600
560 NEXT K
570 NEXT N1
580
590 REM   LOWPASS R1,R2 VALUES
600 LET R1=R0*M/A*(1+SQR(1-A/S/S/M))
610 LET R2=R0*M/A*(1-SQR(1-A/S/S/M))
620 GO TO 700
630
640 REM   HIGHPASS C2,R1,R2 VALUES
650 LET C2=C
660 LET R1=R0
670 LET R2=R0*A/S/S
680
690 REM   PRINTOUT
700 IF I>1 THEN 720
710 PRINT"   COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):"
720 PRINT
730 PRINT"            SECTION";I
740 PRINT"    ","C1=";C1,"R1=";R1
750 PRINT"    ","C2=";C2,"R2=";R2
760
770 REM   FREQ AND GAIN VALUES FOR GRAPH
780 FOR K=0 TO 36
790 IF I>1 THEN 870
800 IF T>1 THEN 830
810 LET F(K)=F/25*(1+K)
820 GO TO 840
830 LET F(K)=F*25/(37-K)
840 LET K7=2-INT(LOG(F(K))/Q1)
850 LET F(K)=INT(F(K)*10↑K7+.5)/10↑K7
860 LET G(K)=0
870 IF T>1 THEN 900
880 LET G=(1-R1*C1*R2*C2*(Q0*F(K))↑2/1E6)↑2+((R1+R2)*C2*Q0*F(K)/1E3)↑2
890 GO TO 910
900 LET G=(1-1/R1/C1/R2/C2/(Q0*F(K))↑2*1E6)↑2+(2/R2/C1/Q0/F(K)*1E3)↑2
910 LET G(K)=G(K)-10*LOG(G)/Q1
920 NEXT K
930 NEXT I
940 GOSUB 4020
950 STOP
2000
2010 REM  HEADING SUBROUTINE
2020 IF R>0 THEN 2050
2030 PRINT"    ","BUTTERWORTH ";
```

however, since a stable unity-gain active element is easy to obtain and most practical filters do not require a high Q (the highest Q in a 10-pole Butterworth filter is less than 5).

In addition the active element is a low-pass filter. It alone will limit the filter response, especially in high-Q sections. An active element cutoff frequency 50 times the section resonant frequency can shift the frequency of the peak response by as much as 5% even for a Q as low as 5.

Synthesize complete filter from sections

$N/2$ filter sections must be cascaded to synthesize an even-pole filter. The component values of each section are computed for a different conjugate pole pair in the desired transfer function. The sections can be assembled in any order provided only that each section is driven from a low-impedance source. Odd-pole filters may also be synthesized[6,7] but single-pole sections are inefficient and three-pole sections are difficult to design. The "RC FIL" computer program therefore considers only even-pole filters.

Check by calculating gain vs frequency

After the N-pole filter has been completely designed the actual frequency response can be compared with the desired response. Compute the actual response analytically by adding the individual section gains (in dB) at each of several test frequencies. For a two-pole, low-pass section, the magnitude of the gain is $L'(j\omega)$:

$$L'(j\omega) = [(1 - R_1C_1R_2C_2\omega^2)^2 + ((R_1+R_2)C_2\omega)^2]^{-1/2}.$$

For a two-pole high-pass section, the magnitude of the gain is $H'(j\omega)$.

$$H'(j\omega) = \left[\left(1 - \frac{1}{R_1C_1R_2C_2\omega^2}\right)^2 + \left(\frac{C_1+C_2}{C_1R_2C_2\omega}\right)^2\right]^{-1/2}$$

In both cases the dB gain is found by taking the common logarithm of the voltage gain and multiplying by 20.

Sample designs illustrate technique

The calculations for designing low-pass or high pass Chebyshev and Butterworth RC filters have been incorporated in the "RC FIL" computer program. The program, listed in Fig. 2, is written in BASIC for use on a time-shared computer.[8]

A BASIC program consists of a series of typed lines, each beginning with a line number followed by a command word. Unless otherwise instructed the computer works on one line at a time in order of increasing line number. To understand a BASIC program, the user must first learn the command words that make up the vocabulary. Some of the command words together with their meanings are listed in Table 3.

Let's examine two applications of the RC FIL program. The first example is a six-pole Butterworth low-pass filter with a 3-dB cut frequency at 1 kHz and a maximum C value of 33 nanofarads. The second is a four-pole Chebyshev high-pass filter with a cut frequency at 1 kHz,

```
2040 GO TO 2060
2050 PRINT"    ","CHEBYSHEV ";
2060 IF T>1 THEN 2090
2070 PRINT"LOWPASS RC FILTER"
2080 GO TO 2100
2090 PRINT"HIGHPASS RC FILTER"
2100 PRINT
2110 PRINT
2120 PRINT
2130 PRINT"    NUMBER OF POLES =";N
2140 PRINT
2150 IF R>0 THEN 2180
2160 PRINT"    3-DB FREQ (KHZ) =";F
2170 GO TO 2210
2180 PRINT"    CUT FREQ (KHZ) =";F
2190 PRINT
2200 PRINT"    PASSBAND RIPPLE (DB) =";R
2210 PRINT
2220 PRINT
2230 RETURN
4005
4010 REM    GRAPH SUBROUTINE
4020 FOR I=1 TO 53-2*N
4030 PRINT
4040 NEXT I
4050 GOSUB 2020
4060 LET I4=1
4070 IF R>0 THEN 4090
4080 LET I4=2
4090 PRINT"    ","                    INSERTION GAIN (DB)"
4100PRINT"-15        -12        -9        -6        -3        0        +3
4110PRINT" + - - - - + - - - - + - - - - + - - - - + - - - - + - - - - +
4120 FOR I=0 TO 36 STEP I4
4130 LET G=INT(G(I)/.3+.5)+51
4140 FOR I1=0 TO 20
4150 LET I2=G-3*I1
4160 IF I2<0 THEN 4260
4170 IF I2>2 THEN 4260
4180 IF I2<2 THEN 4210
4190 PRINT"  *";
4200 GO TO 4370
4210 IF I2<1 THEN 4240
4220 PRINT" * ";
4230 GO TO 4370
4240 PRINT"*  ";
4250 GO TO 4370
4260 IF INT(I/6)<I/6 THEN 4300
4270 IF INT((I1+7)/10)=INT(I1+7)/10 THEN 4320
4280 IF INT(I1/10)=I1/10 THEN 4340
4290 IF INT((I1+3)/10)=INT(I1+3)/10 THEN 4360
4300 PRINT"  . ";
4310 GO TO 4370
4320 PRINT"  +";
4330 GO TO 4370
4340 PRINT" + ";
4350 GO TO 4370
4360 PRINT"+  ";
4370 NEXT I1
4380 PRINT F(I)
4390 IF R>0 THEN 4410
4400 PRINT
4410 NEXT I
4420 PRINT"   ","   ","   ","          FREQUENCY (KHZ)"
4430 RETURN
4440 END
```

2. **This BASIC program** finds the component values for the RC-active filter specifications entered as "DATA" in line 0.

Table 3. BASIC commands

Type	Word	Function
Nonexecutable	REM	Allows the insertion of remarks in the program listing
	DIM	Reserves extra memory room for large variable arrays
	DATA	Stores numerical data to be used in the problem solution
Input/Output	READ	Obtains numerical data from DATA statements
	PRINT	Types output statements and numerical answers
Computational	LET	Computes variable values according to algebraic formulas
Sequencing	GO TO	Alters the normal order of computation
	IF. . .THEN	Conditionally alters the order of computation
	FOR. . .TO NEXT	Causes the intervening commands to be repeated several times
	GO SUB RETURN	Routes computation to and from a subroutine (subsection) of the program
Termination	STOP	Stops computation (at any point in the program)
	END	Stops computation (this must be the last sequential command in a program)

Table 5. Variables used in program

Name	Definition
T	Type of filter (1 = Low-pass, 2 = High-pass)
N	Number of poles
F	Cut frequency (in kHz)
R	Chebyshev passband ripple (in dB)
C	Maximum circuit capacitance (in nanofarads)
Qϕ	2π (phase conversion constant from radians to degrees)
Q1	ln 10 (gain conversion constant from natural logs to common logs)
I	Iteration index (I = 1 to N/2) for the 2-pole sections
F (37)	Frequency values (independent variable)
G (37)	Gain values (dependent variable)
C (25)	Standard capacitance values per decade
X1	Minor Chebyshev ellipse radius
X2	Major Chebyshev ellipse radius
R	Chebyshev ripple factor
D	Intermediate Chebyshev parameter
P	Root location phase angle
S	Real component (σ) of pole location
W	Imaginary component (ω) of pole location
A	Squared magnitude of pole location
Rϕ	Nominal resistance level
R1,R2,C1,C2	Component values (kilohms and nanofarads)
M	Ratio of C1/C2 in low-pass sections
K6	Number of standard capacitance values per decade
K,N1,I1	Iteration loop indices
N2,K7	Normalization constants
G,I2	Intermediate variables used in graph routine

Table 4. Program block outline

Line numbers	What is accomplished
0 – 90	Data input and instructions to user
100 – 120	Constant definition and variable dimensioning
130 – 170	Heading print-out (using subroutine 2000 – 2230)
180 – 670	Calculations for each 2-pole section
220 – 410	Calculation of root pair locations
420 – 620	Calculation of low-pass component values
630 – 670	Calculation of high-pass component values
680 – 750	Component value print-out
770 – 930	Selection of frequency values and calculations of gain values for graph
940 – 950	Graph print-out (using subroutine 4000 – 4440)
2000 – 2230	Print subroutine for page headings
4000 – 4440	Print subroutine for graph

a passband ripple of 2.4 dB and a maximum C value of 10 nF.

To solve the first filter design, we must retype line 0 as follows:

0 DATA 1, 6, 1, 0, 33.

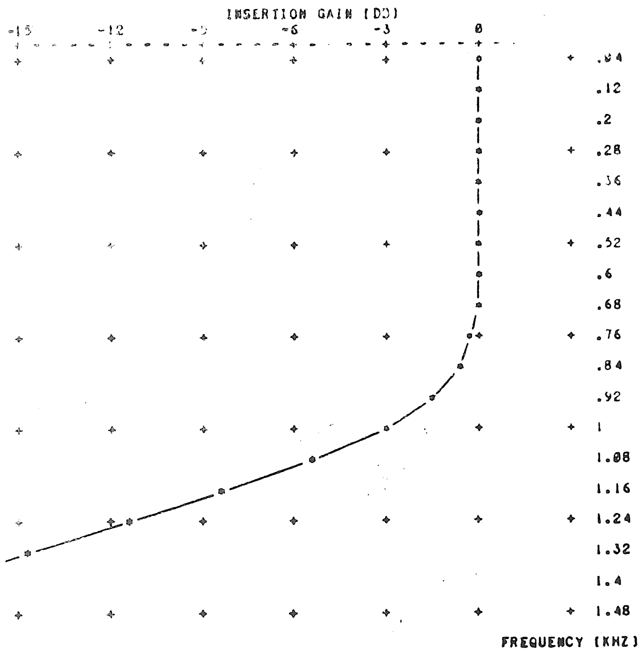
The remark ("REM") statements in Fig. 2 explain how the desired filter specifications are entered into the program (see Table 4). The variables used are listed in Table 5. When the program is run with this "DATA" line, a two-page print-out (Fig. 3) is generated. Each page begins with a descriptive heading. Figure 3A shows the first page, which gives the component values, and Fig. 3B the second page, containing a graph of gain versus frequency.

For low-pass designs, the C_1 value in each of the sections is set equal to the maximum C value specified in the DATA line. In this case C_1 is 33 nF in each of the three sections. The C_2 value is then chosen as the largest value that can be realized from a standard decade list of C values

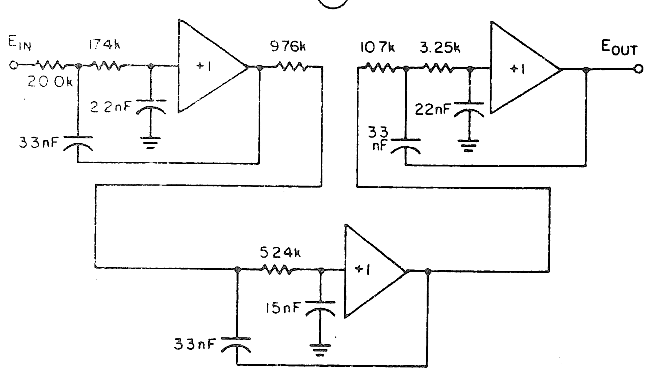
```
BUTTERWORTH LOWPASS RC FILTER

NUMBER OF POLES : 6
3-DB FREQ (KHZ) : 1

COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):

     SECTION 1
          C1: 33            R1: 20.8192
          C2: 2.2           R2: 17.4284

     SECTION 2
          C1: 33            R1: 9.76477
          C2: 15            R2: 5.24051

     SECTION 3
          C1: 33            R1: 10.7214
          C2: 22            R2: 3.25427
```

(A)

```
NUMBER OF POLES : 6
3-DB FREQ (KHZ) : 1
```



(B)

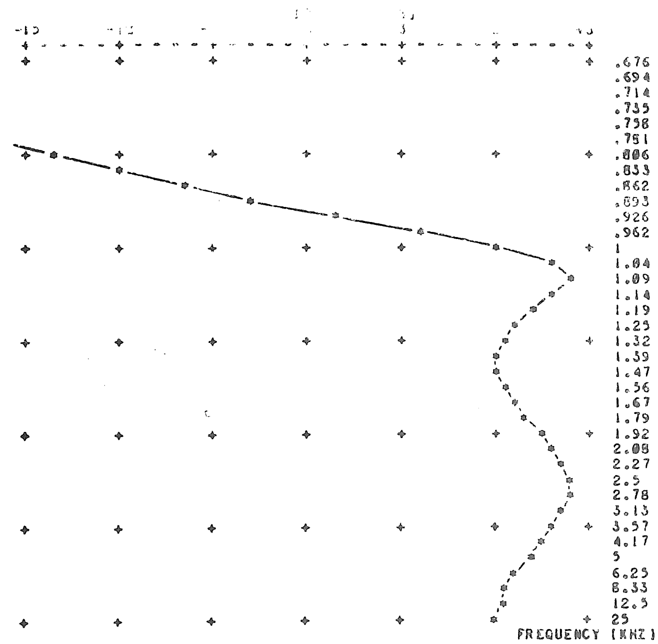


(C)

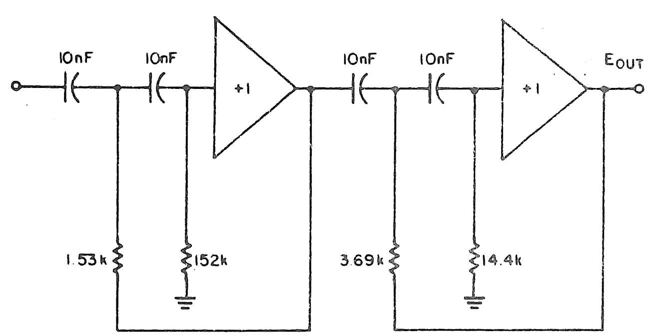
3. **Computer printout** of component values (A) and frequency response curve (B) for six-pole Butterworth lowpass filter (C) is shown above.

```
CHEBYSHEV HIGHPASS RC FILTER

NUMBER OF POLES : 4
CUT FREQ (KHZ) : 1
PASSBAND RIPPLE (DB) : 2.4

COMPONENT VALUES (C IN NANOFARADS, R IN KILOHMS):

     SECTION 1
          C1: 10            R1: 1.52734
          C2: 10            R2: 151.987

     SECTION 2
          C1: 10            R1: 3.68733
          C2: 10            R2: 14.3802
```

```
NUMBER OF POLES : 4
CUT FREQ (KHZ) : 1
PASSBAND RIPPLE (DB) : 2.4
```



(B)



(C)

4. **Computer printout** of component values (A) and frequency response (B) for four-pole Chebyshev high-pass filter (C) is shown above.

(contained in line 450 of the program, Fig. 2). The list may be changed, as required, to reflect available C values. The first number in the list (in this case, 6) is the number of values in the list. The following numbers, are the C values arranged in increasing order of magnitude. In the print-out (Fig. 3A) the values of C_2 for the three sections are 2.2 nF, 15 nF and 22 nF. The R values, which are computed from these C values and the filter specifications, are non-

Deriving the fundamental filter equations

Modern filter design theory is based upon an analysis of the filter transfer function, the ratio of output to input in the complex frequency plane, the s-plane. Attention is centered on the location of the "poles" of the transfer function. The poles are the values of s for which the denominator of the transfer function vanishes, or is equal to zero.

The poles of the normalized Butterworth low-pass function are equally spaced on a unit circle centered at the origin of the s-plane.[9-12] For an N-pole function, the phase angles, P_i, of the pole locations are:

$$P_i = \frac{\pi}{2} + \frac{\pi}{2N}(2i-1),\ i = 1 \text{ to } N. \quad (1)$$

The real (σ_i) and imaginary $(j\omega_i)$ Cartesian coordinates of the pole locations are therefore:

$$\sigma_i = \cos(P_i) \quad (2a)$$

$$j\omega_i = j\sin(P_i) \quad (2b)$$

The poles of the normalized Chebyshev low-pass function are unequally spaced on an ellipse centered at the origin of the s-plane.[9-12] The minor radius $X1$ of the ellipse is parallel to the real (σ) axis, and the major radius $X2$ is parallel to the imaginary $(j\omega)$ axis of the s-plane. The values of $X1$ and $X2$ depend on the peak-to-peak ripple R (in dB) of the voltage waveform in the passband, and on the number of poles, N. It is convenient to define a ripple factor, ϵ, in terms of R according to the following convention:

$$\epsilon \triangleq (10^{R/10} - 1)^{1/2}. \quad (3)$$

If we use this definition then $X1$ and $X2$, the radii of the ellipse, are:

$$X1 = \sinh\left[\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right] \quad (4a)$$

$$X2 = \cosh\left[\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]. \quad (4b)$$

Since it is inconvenient to work with hyperbolic functions, the standard mathematical relations[13]

$$\sinh x = (e^x - e^{-x})/2 \quad (5a)$$

$$\cosh x = (e^x + e^{-x})/2 \quad (5b)$$

$$\sinh^{-1} x = \ln[x + (x^2+1)^{1/2}] \quad (5c)$$

will be used to obtain the more convenient form:

$$X1 = [D - (1/D)]/2 \quad (6a)$$

$$X2 = [D + (1/D)]/2, \quad (6b)$$

where

$$D = \left[\frac{1}{\epsilon} + \left(\frac{1}{\epsilon^2} + 1\right)^{1/2}\right]^{1/N} \quad (7)$$

The real and imaginary components of the pole locations for both types of filters can therefore be expressed in terms of ONE set of equations:

$$\sigma_i = X1\cos(P_i) \quad (8a)$$

$$j\omega_i = jX2\sin(P_i) \quad (8b)$$

where P_i is defined in Eq. 1.

If the number of poles is even, an N-pole function may be factored into $N/2$ conjugate pairs, since each pole, s_i, has a conjugate pole s_{N+1-i}. Let

$$A_i \triangleq (\sigma_i + j\omega_i)(\sigma_i - j\omega_i) = \sigma_i^2 + \omega_i^2. \quad (9)$$

The normalized transfer function $L_i(s)$ of a low-pass pole pair is then:

$$L_i(s) = \frac{\frac{1}{A_i}}{\frac{1}{A_i}s^2 - \frac{2\sigma_i}{A_i}s + 1} \quad (10)$$

standard.

The second page of the print-out (Fig. 3B) is a plot of the logarithmic gain in decibels versus linear frequency in kilohertz. Plus signs are used to form the graph grid, and asterisks indicate the data points. The lines connecting the asterisks were drawn in after the print-out was completed to make the response curve more legible. The curve in Fig. 3B agrees with the standard six-pole Butterworth characteristic.[9]

The complete circuit schematic for the six-pole low-pass filter is shown in Fig. 3C. Three basic low-pass circuits have been cascaded (Fig. 1), with the component values of each section determined from Fig. 3A. A laboratory model of the circuit was built using the last active element shown in Table 1. Tests showed that it did have the frequency response predicted in Fig. 3B.

In the example of the four-pole Chebyshev high-pass filter, line 0 in the computer program (Fig. 2) is in the correct format for solving the filter design. The solution is shown in Fig. 4. Figure 4A gives the component values, Fig. 4B the frequency response, and Fig. 4C the complete circuit schematic.

The headings provided in the computer print-out (Figs. 4A and 4B) for the Chebyshev filter are similar to those for the Butterworth case, but they include the definition of an additional parameter—the passband ripple. The plot of Fig. 4B shows every data point (rather than every other point as in Fig. 3B), to more clearly define where the constant factor $1/A_i$ in Eq. 10 normalizes the maximum value of $L_i(s)$ to unity gain. Since the low-pass section (Fig. 1) must have unity gain at dc, the factor $1/A_i$ cannot be accommodated by Eq. 12 for the Chebyshev case. Chebyshev filters designed in this way will have unity gain ripple minimums rather than the more [illegible] restriction also holds for high-pass design.

The normalized transfer function $H_i(s)$ of a high-pass pole pair is obtained by replacing the variable s in (10) by $1/s$:

$$H_i(s) = \frac{\frac{1}{A_i}}{\frac{1}{A_i s^2} - \frac{2\sigma_i}{A_i s} + 1} \qquad (11)$$

The transfer function $L'(s)$ of the two-pole low-pass RC filter section (Fig. 1) is

$$L'(s) = \frac{1}{R_1C_1R_2C_2s^2 + (R_1 + R_2)\,C_2s + 1}. \qquad (12)$$

To realize $L_i(s)$ with $L'(s)$, Eqs. 10 and 12 must be equated, term by term. If the constant factor is neglected, this leads to two equations in four unknowns, R_1, R_2, C_1 and C_2:

$$R_1C_1R_2C_2 = 1/A_i \qquad (13a)$$

$$(R_1 + R_2)\,C_2 = -2\,\sigma_i/A_i. \qquad (13b)$$

If C_1 and C_2 are selected as standard values,

$$C_1 = C \qquad (14a)$$

$$C_2 = C/M,\ M > 1 \text{ for realizability,} \qquad (14b)$$

then the normalized values of R_1 and R_2 are determined from

$$R_1 = \frac{-\sigma_i M}{A_i C}\left[1 + \left(1 - \frac{A_i}{\sigma_i^2 M}\right)^{1/2}\right] \qquad (15a)$$

$$R_2 = \frac{-\sigma_i M}{A_i C}\left[1 - \left(1 - \frac{A_i}{\sigma_i^2 M}\right)^{1/2}\right] \qquad (15b)$$

From Eq. 15 it can be seen that the value of M selected in Eq. 14b must satisfy the inequality

$$M \geq A_i/\sigma_i^2. \qquad (16)$$

The [illegible] standard value of C_2 may be found by first choosing $C_2 > C$ and then trying successively smaller standard values for C_2 until Eq. 16 is satisfied.

The transfer function $H'(s)$ of the two-pole high-pass RC filter (Fig. 1) section is

$$H'(s) = \frac{1}{\frac{1}{R_1C_1R_2C_2s^2} + \frac{C_1 + C_2}{C_1R_2C_2s} + 1} \qquad (17)$$

As in the low-pass case, $H_i(s)$ may be realized with $H'(s)$ by selecting standard C values and then computing the normalized R values. The latter are derived by equating the denominators of Eqs. 11 and 17:

$$C_1 = C \qquad (18a)$$

$$C_2 = C \qquad (18b)$$

$$R_1 = \frac{-\sigma_i}{C} \qquad (19a)$$

$$R_2 = \frac{-A_i}{\sigma_i C} \qquad (19b)$$

The component value formulas are summarized in Table 2.

the faster-moving Chebyshev response.

The standard decade list of C values is not utilized in the high-pass design (Fig. 4A) since both C_1 and C_2 are set equal to the maximum C value specified in the DATA line. The high-pass graph (Fig. 4B) uses an inverse-linear frequency scale to emphasize the inverse symmetry of the high-pass design with respect to its low-pass prototype. The two filter sections of the complete schematic (Fig. 4C) use the basic high-pass rather than the low-pass circuit. ■■

References:

1. Shirley, Fredrick R., "You Don't Have to be a Programmer," *Electronic Design*, April 12, 1967, pp. 54-60.
2. *BASIC Language Reference Manual*, General Electric Information Service Publication 202026A, April 1968.
3. Strunk, B., "Designing an Active Filter," *EEE*, October 1965, pp. 60-61.
4. Melsheimer, Robert S., "If You Need Active Filters," *Electronic Design*, April 12, 1967, pp. 78-82.
5. Sun, H. H. *Synthesis of RC Networks*, Hayden Book Co., Inc., New York, N. Y., 1967, pp. 106-112.
6. Kincaid, Russell, "RC Filter Design by the Numbers," *The Electronic Engineer*, October 1968, pp. 57-64.
7. Mollinga, Thomas, "Active Bandpass Filters," *EEE*, August, 1966, pp. 115-119.
8. *Command System Reference Manual*, General Electric Information Service Publication 229116A, April 1968.
9. Eggen, C. P. and McAllister, A. S., "Modern Low-Pass Filter Characteristics," *Electro-Technology*, August 1966, pp. 50-55.
10. Balabanian, Norman, *Network Synthesis*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1958, pp. 381-411.
11. Guillemin, Ernst A., *Synthesis of Passive Networks*, John Wiley & Sons, Inc., New York, N. Y., 1957, pp. 587-602.
12. Kole, William C., "Fewer Sections and Better Specs," *Electronic Design*, Feb. 15, 1968, pp. 84-89.
13. *CRC Standard Mathematical Tables*, Chemical Rubber Publishing Co., Cleveland, O., 1959, pp. 427-432.

Tape #1.

```
*
*
*
*
*
*E A        Erase All
* @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
*C     RC ACTIVE FILTER PROGRAM BY KINCAID AND SHIRLEY
*C     MODIFIED FOR FOCAL BY TOM BEAN & WILLIAM ROMANS
*C            T:  TYPE OF FILTER (ENTER L FOR LOWPASS;H FOR HIGHPASS)
*C            N:  NUMBER OF POLES (MUST BE EVEN NUMBER)
*C            H:  CUTOFF FREQUENCY IN KHZ
*C            R:  ALLOWABLE RIPPLE IN PASSBAND(Ø FOR BUTTERWORTH DESIGN)
*C            C:  MAXIMUM CAPACITANCE VALUE IN NANOFARADS
*C     R1,R2  GIVEN IN KOHMS
*
*C-FOCAL,1969
*
*Ø1.Ø2  Ask  !? T N H R C1 ? ,!!;S PI=3.14159;S C(6)=1;S C(Ø)=1Ø
*Ø1.Ø4  Set  C(1)=6.8;S C(2)=4.7;S C(3)=3.3;S C(4)=2.2;S C(5)=1.5
*Ø1.Ø6  For  I=1,N/2;D 3
*
*Ø3.Ø8  If   (-R)3.16;S X1=1;S X2=1;G 3.28
*Ø3.16  Set  E=FSQT(FEXP(R*2.3Ø259/1Ø)-1)
*Ø3.18  S    D=FEXP(FLOG(1/E+FSQT(1/E↑2+1))/N)
*Ø3.2Ø  S    X1=(D-1/D)/2;S X2=(D+1/D)/2
*Ø3.28  S    S=X1*FCOS(PI/2+PI*(2*I-1)/2*N)
*Ø3.3Ø  S    W=X2*FSIN(PI/2+PI*(2*I-1)/2*N)
*Ø3.32  S    A=S↑2+W↑2
*Ø3.34  S    X3=2*PI*H*C1
*Ø3.36  If   (1-12/T)7.Ø4;F N1=-1,9;D 4
*
*Ø4.Ø4  S    P=FITR(FLOG(C1)/2.3Ø259)-N1;F K=1,6;D 5
*
*Ø5.Ø2  If   (-P)5.Ø8
*Ø5.Ø4  S    C2=C(K)/1Ø↑(-P);S C3=C(K-1)/1Ø↑(-P);G 5.12
*Ø5.Ø8  S    C2=C(K)*1Ø↑P;S C3=C(K-1)*1Ø↑P
*Ø5.12  S    M=C1/C2;I (M-A/S↑2)5.16;D 6
*Ø5.16  Return
*
*Ø6.Ø4  Set  R1=(-S*1Ø↑3/(X3))*(M/A)*(1+FSQT(1-A/(S↑2*M)))
*Ø6.Ø6  Set  R2=(-S*1Ø↑3/(X3))*(M/A)*(1-FSQT(1-A/(S↑2*M)))
*Ø6.Ø8  If   (A/S↑2-C1/C3)6.1Ø;D 8
*Ø6.1Ø  Return
*
*Ø7.Ø4  Set  C2=C1;S R1=-S*1Ø↑3/(X3);S R2=(-S*1Ø↑3/(X3))*A/S↑2;D 8
*Ø7.Ø6  Set  R2=(-S*1Ø↑3/(X3))*A/S↑2;D 8
*
```

```
*Ø8.1Ø If    "C1,C2,R1,R2";T %8.Ø5,C1,C2,R1,R2,!!
*Ø8.14 If  (N/2-1)9.Ø1,9.Ø1,8.16
*Ø8.16 Return
*
*Ø9.Ø1 Quit
**@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
*Erase
*Go
                                                        Example
T :L N :6 H :1 R :Ø C1 :33
Low  # Sect's  f cutoff  Ripple %  Cmax in nf
SEC.=  1
C1,C2,R1,R2=  33.ØØØØØ=   2.2ØØØØ=   2Ø.Ø1779=   17.4298Ø

SEC.=  2
C1,C2,R1,R2=  33.ØØØØØ=   15.ØØØØØ=   9.76469=   5.24Ø55

SEC.=  3
C1,C2,R1,R2=  33.ØØØØØ=   22.ØØØØØ=   1Ø.7214Ø=   3.25429

*
*Erase
*Go

T :H N :4 H :1 R :2.4 C1 :1Ø nf
High  # Sects  f cutoff  Ripple %  Cmax
SEC.=  1
C1,C2,R1,R2=  1Ø.ØØØØØ=   1Ø.ØØØØØ=   1.52734=   151.988ØØ

SEC.=2
C1,C2,R1,R2=  1Ø.ØØØØØ   1Ø.ØØØØØ=   3.68732=   14.38Ø4Ø

*                                                       Start Tap #2
*E A        Erase All
* @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
*C  GAIN VERSUS FREQUENCY PLOT FOR RC FILTER
*C  RC  ACTIVE FILTER PROGRAM BY KINCAID AND SHIRLEY
*C  MODIFIED FOR FOCAL BY TOM BEAN & WILLIAM E ROMANS
*C  INPUT: T  IS TYPE OF FILTER (ENTER L FOR LOWPASS;H FOR HIGHPASS)
*C         N IS NO. OF POLES (MUST BE EVEN)
*C         H IS CUTOFF FREQUENCY IN KHZ
*C         R IS PASSBAND RIPPLE IN DB (Ø FOR BUTTERWORTH)
*C         C1 IS MAXIMUM CAPACITANCE IN NANOFARADS
*C  RE-ENTER VALUES FOR THE ABOVE WHICH WERE USED IN CALCULATING
*C     R1  R2  C1  C2  AND ENTER THE CALCULATED VALUES FOR C2  R1  AND  R2
*
```

```
*C-FOCAL, 1969
*
*Ø1.Ø5  Ask !  ?T  N  H  R  C1  ?,!!;Set J=N/2;For I=1,J;Ask ?C2  R1  R2  ?,!;Do 1.15
*Ø1.1Ø  Go 1.2Ø
*Ø1.15  Set T=12/T;Set X(I)=R1*R2*C1*C2;Set Y(I)=(R1+R2)*C2;Set Z(I)=R2*C1
*Ø1.2Ø  Type !!;Type"INSERTION GAIN (DB)",!!
*Ø1.25  For KX=Ø,6;Type%2,-15+3*KX;Type"      "    — abscissa scale
*Ø1.3Ø  Type !;For KZ=1,6;Type" + - - - - "
*Ø1.35  Type " + ",!
*Ø1.4Ø  For K=Ø,36;Do 2;Do 4
*Ø1.45  Quit
*
*Ø2.Ø2  If  (1-T)2.Ø6;Set HK=H*(1+K)/25;Go 2.14
*Ø2.Ø6  Set HK=25*H/(37-K)
*Ø2.14  Set B=2*3.14159*HK;For I=1,J;Do 3
*
*Ø3.Ø5  If  (1-T)3.15;
*Ø3.1Ø  Set GX(I)=(1-X(I)*B↑2/1Ø↑6)↑2+(Y(I)*B/1Ø↑3)↑2;Go 3.2Ø
*Ø3.15  S   GX(I)=(1-1Ø↑6/(X(I)*B↑2))↑2+(2*1Ø↑3/(Z(I)*B))↑2
*Ø3.2Ø  S   G(I)=-1Ø*FLOG(GX(I))/2.3Ø259
*
*Ø4.Ø4  Set G=FITR((G(1)+G(2)+G(3)+G(4)+G(5))/.3+.5)+51;For I1=Ø,2Ø;Do 5
*Ø4.Ø6  Do 6
*
*Ø5.Ø4  Set I2=G-3*I1;If(I2)5.22;If(I2-2)5.12,5.Ø8,5.22
*Ø5.Ø8  Type "  *";Return
*Ø5.12  If  (I2-1)5.18;Type" *  ";Return
*Ø5.18  Type "*   ";Return
*Ø5.22  If  (FITR(K/6)-K/6)5.3Ø;
*Ø5.24  If  (FITR((I1+7)/1Ø)-FITR(I1+7)/1Ø)5.26,5.32;
*Ø5.26  If  (FITR(I1/1Ø)-I1/1Ø)5.28,5.34;
*Ø5.28  If  (FITR((I1+3)/1Ø)-FITR(I1+3)/1Ø)5.3Ø,5.36;
*Ø5.3Ø  Type "    ";Return
*Ø5.32  Type "   +";Return
*Ø5.34  Type " +  ";Return
*Ø5.36  Type "+   ";Return
*
*Ø6.Ø2  Type %4.Ø3, HK,!
** @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
*Erase
*Go                                                        Prob #1

T :L  N :6  H :1  R :Ø  C1 :33

C2 :2.2  R1 :2Ø.Ø177  R2 :17.4298
C2 :15  R1 :9.764  R2 :5.24
C2 :22  R1 :1Ø.7214  R2 :3.25429
```

INSERTION GAIN (DB)

```
=-15  =-12  =- 9  =- 6  =- 3  =  Ø  =  3
+-----+-----+-----+-----+-----+-----+
+     +     +     +     +     *     + = Ø.Ø4Ø
                              *       = Ø.Ø8Ø
                              *       = Ø.12Ø
                              *       = Ø.16Ø
                              *       = Ø.2ØØ

+     +     +     +     +     *     + = Ø.28Ø
                              *       = Ø.32Ø
                              *       = Ø.36Ø
                              *       = Ø.4ØØ
                              *       = [illegible]
                              *       = Ø.48Ø
+     +     +     +     +     *     + = Ø.52Ø
                              *       = Ø.56Ø
                              *       = Ø.6ØØ
                              *       = Ø.64Ø
                              *       = Ø.68Ø
                              *       = Ø.72Ø
+     +     +     +     +     *     + = Ø.76Ø
                              *       = Ø.8ØØ
                             *        = Ø.84Ø
                            *         = Ø.88Ø
                           *          = Ø.92Ø
                          *           = Ø.96Ø
+     +     +     +     +*    +     + = 1.ØØØ
                      *               = 1.Ø4Ø
                    *                 = 1.Ø8Ø
                *                     = 1.12Ø
             *                        = 1.16Ø
           *                          = 1.2ØØ
+     + *   +     +     +     +     + = 1.24Ø
     *                                = 1.28Ø
 *                                    = 1.32Ø
                                      = 1.36Ø
                                      = 1.4ØØ
                                      = 1.44Ø
+     +     +     +     +     +     + = 1.48Ø
*
*Erase
*Go
```

T :H N :4 H :1 R :2.4 C1 :1Ø   Problem #2.

C2 :1Ø R1 :1.527 R2 :151.98
C2 :1Ø R1 :3.68 R2 :14.38

INSERTION GAIN (DB)

```
=-15  =-12  =- 9  =- 6  =- 3  =  Ø  =  3
 +-----+-----+-----+-----+-----+-----+
 +     +     +     +     +     +     + = Ø.676
                                       = Ø.695
                                       = Ø.714
                                       = Ø.735
                                       = Ø.758
                                       = Ø.781
 + *   +     +     +     +     +     + = Ø.8Ø7
        *                              = Ø.833
           *                           = Ø.862
               *                       = Ø.893
                     *                 = Ø.926
                           *           = Ø.962
 +     +     +     +     +     *     + = 1.ØØØ
                                  *    = 1.Ø42
                                   *   = 1.Ø87
                                  *    = 1.136
                                 *     = 1.191
                                *      = 1.25Ø
 +     +     +     +     +     *     + = 1.316
                              *        = 1.389
                              *        = 1.471
                               *       = 1.563
                                *      = 1.667
                                *      = 1.786
 +     +     +     +     +     + *   + = 1.923
                                  *    = 2.Ø83
                                   *   = 2.273
                                    *  = 2.5ØØ
                                    *  = 2.778
                                   *   = 3.125
 +     +     +     +     +     +  *  + = 3.572
                                 *     = 4.167
                                 *     = 5.ØØØ
                                *      = 6.25Ø
                                *      = 8.333
                                *      = 12.5Ø
 +     +     +     +     +     *     + = 25.ØØ
 *
```